$T + V = h$ (Constant)

P.G. sem - III
Paper - XIII
Unit-2nd. classical mechanics

Hamiltonian or Hamilton's function

A function $H(p_i, q_i, t)$ defined as $H = \sum_{i=1}^{n} p_i \dot{q}_i - L$

where $p_i = \frac{\partial L}{\partial \dot{q}_i}$ is known as Hamiltonian function or Hamilton's function

Note: $p_i = \frac{\partial L}{\partial \dot{q}_i}$ is called generalised component of linear momentum.

Scanned with Cam

$$\Rightarrow H = \sum_{i=1}^{n} \frac{\partial(T_2+T_1+T_0)}{\partial \dot{q}_i}\dot{q}_i - T + V$$

$$\Rightarrow H = 2T_2 + \cancel{T_1} - T_2 - \cancel{T_1} - T_0 + V$$

$$= T_2 - T_0 + V$$

$$= T_2 + T_1 + T_0 + V - T_1 - 2T_0$$

$$\Rightarrow H = T + V - T_1 - 2T_0$$

$$\Rightarrow \frac{dH}{dt} = \frac{d}{dt}(T+V) - \frac{d}{dt}(T_1 + 2T_0) \quad \text{— (1)}$$

Again $H = \sum_{i=1}^{n} p_i \dot{q}_i - L$

$$\Rightarrow \frac{dH}{dt} = \sum_{i=1}^{n} \dot{p}_i \dot{q}_i + \sum_{i=1}^{n} p_i \ddot{q}_i - \sum_{i=1}^{n} \frac{\partial L}{\partial q_i}\dot{q}_i - \sum_{i=1}^{n} \frac{\partial L}{\partial \dot{q}_i}\ddot{q}_i - \frac{\partial L}{\partial t}$$

$$\Rightarrow \frac{dH}{dt} = \cancel{\sum_{i=1}^{n} \dot{p}_i \dot{q}_i} + \cancel{\sum_{i=1}^{n} p_i \ddot{q}_i} - \cancel{\sum_{i=1}^{n} \dot{p}_i \dot{q}_i} - \cancel{\sum_{i=1}^{n} p_i \ddot{q}_i} - \frac{\partial L}{\partial t} \quad \text{as } \dot{p}_i = \frac{\partial L}{\partial q_i}$$

$$\Rightarrow \frac{dH}{dt} = -\frac{\partial L}{\partial t}$$

putting this value in (1), we get—

$$\frac{d}{dt}(T+V) = \frac{d}{dt}(T_1 + 2T_0) - \frac{\partial L}{\partial t}$$

Integrating it with respect to t, we get

$$T + V = T_1 + 2T_0 - \int \frac{\partial L}{\partial t}\,dt + h$$

This is the energy integral for holonomic system. If the system is conservative then

(i) The system is scleronomic

(ii) V does not contain t explicitly

then $T_1 = T_0 = \frac{\partial L}{\partial t} = 0$

Then the energy integral may be written as $T + V = h$ (constant).

Scanned with Cam